Unstable Motivic Homotopy

David Zhu

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Goal

Design a homotopy theory for schemes.

We will be working with Sm/S, the category of smooth schemes of finite type over S, a Noetherian scheme of finite dimension.

$S = k$

Question

What do we mean by a "homotopy theory"?

A "homotopy theory" could be any one of the two structures

1. A simplicial model category (Morel-Voevodsky) Late 1990s
2. An ∞-category (What cool kids do these days)

Model Cat $\Longrightarrow$ ∞-cat

Spoilers

There are multiple ways to set up motivic spaces (model categorical and ∞-categorical), but all of them produce the same homotopy theory. We will use the ∞-categorical version following [1].

Definition

The ∞-category of **motivic spaces** $Spc(S)$ is

$$Spc(S) = \mathrm{Shv}_{\mathrm{Nis}}(Sm/S) \cap \mathrm{PShv}_{\mathbb{A}^1}(Sm/S) \subseteq \mathrm{PShv}(Sm/S)$$

which is the fullsubcategory of presheaves that are Nisnevich sheaves and are $\mathbb{A}^1$-local.

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Setups

Model cat requires bicompleteness.

Small obj argument $\rightsquigarrow$ cell colimit

Problem 1

The category Sm/S is not cocomplete.

In particular, we do not have all pushouts, so we cannot always glue schemes along arbitrary maps, or form quotients, unlike in the category of spaces.

Example: $G \curvearrowright X$ $\quad X/G$ does not always exist

Preliminary Solution

We consider the category of **presheaves**, denoted by

$$\mathrm{PShv}(Sm/S)$$

This is the functor category consisting of objects

$$\mathcal{F} : (Sm/S)^{op} \to Set$$

For the moment, we can think presheaf of sets, although we will move to presheaf of simplicial sets later on.

Fact 1

Sm/S embeds fully faithfully in $\mathrm{PShv}(Sm/S)$.

This is via the Yoneda embedding.

$X \mapsto h_X \qquad h_X := Hom(-, X)$

Fact 2

$\mathrm{PShv}(Sm/S)$ is bicomplete, and the (co)limits are computed pointwise.

– Pshv (Sm/S) is actually "universal" cocompletion

Problem 3

Yoneda embedding does not preserve existing colimits.

Example

We have the following pushout square in *Sm*/*k*:

$$\begin{tikzcd}
\mathbb{A}^1 - 0 \arrow[r] \arrow[d] & \mathbb{A}^1 \arrow[d] \\
\mathbb{A}^1 \arrow[r] \arrow[ur, phantom, very near end, "\ulcorner"] & \mathbb{P}^1
\end{tikzcd}$$

But the represented presheaves do NOT form a pushout in $\mathrm{PShv}(Sm/S)$.

$$\begin{tikzcd}
\mathrm{Hom}(\overset{\mathbb{P}^1}{-}, \mathbb{A}^1 - 0) \arrow[r] \arrow[d] & \mathrm{Hom}(\overset{\mathbb{P}^1}{-}, \mathbb{A}^1) \arrow[d] \\
\mathrm{Hom}(\overset{\mathbb{P}^1}{-}, \mathbb{A}^1) \arrow[r] & \mathrm{Hom}(\overset{\mathbb{P}^1}{-}, \mathbb{P}^1)
\end{tikzcd}$$

Sheafification

Embedding into the presheaf category loses some "geometry". We would like to preserve some colimits, like the pushout square in the previous example.

Slogan

A Grothendieck topology specifies a class of colimits we want to preserve, and sheafification with respect to the topology is the universal way to do it.

Suppose we have a covering $\{U_i \to X\}_{i \in I}$, this gives rise to a colimit

$$\textstyle\coprod_{ij} U_{ij} \rightrightarrows \coprod_k U_k \to X$$

and a sheaf F will still "recognize" this colimit:

$$\mathrm{PShv}(X, F) \to \mathrm{PShv}(\coprod_k U_k, F) \rightrightarrows \mathrm{PShv}(\coprod_{ij} U_{ij}, F)$$

Yoneda lemma

is equivalent to

$$F(X) \to \coprod_k F(U_k) \rightrightarrows \coprod_{ij} F(U_{ij})$$

sheaf condition

being a colimit.

Fact

A Grothendieck topology is called **subcanonical** if all represented presheaves are sheaves. Zariski, Étale, Nisnevich topology are all subcanonical.

In particular, the Yoneda embedding factors through sheaves

$$\begin{tikzcd}
Sm/S \arrow[rr] \arrow[dr] & & \mathrm{PShv}(Sm/S) \\
& \mathrm{Shv}_{Nis}(Sm/S) \arrow[ur] &
\end{tikzcd}$$

and the following diagram

$$\begin{tikzcd}
\mathrm{Hom}(-, \mathbb{A}^1 - 0) \arrow[r] \arrow[d] & \mathrm{Hom}(-, \mathbb{A}^1) \arrow[d] \\
\mathrm{Hom}(-, \mathbb{A}^1) \arrow[r] & \mathrm{Hom}(-, \mathbb{P}^1)
\end{tikzcd}$$

is a pushout of sheaves in all three topologies above.

Why Nisnevich

Remark: étale could be used for non-smooth Sch
h-topology Voevodsky

Nisnevich topology is the standard choice for motivic homotopy theory.

Theorem (Morel-Voevodsky Purity)

Suppose $Y \to X$ is a closed immersion in Sm/S. Then, there is a motivic equivalence

$$\frac{X}{X \setminus Y} \cong Th_Y(N_{Y/X})$$

Theorem

Algebraic K-theory is a Nisnevich Sheaf, but not étale. (Thomas – Trobaugh)

These are the two serious reasons why we need Nisnevich instead of other topologies.

Sheaves with Homotopy Type

Question

How do we associate a (pre)sheaf a homotopy type?

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How do we associate a (pre)sheaf a homotopy type?

The category **Set** is cocomplete, so taking the functor category into **Set** is theuniversal cocompletion; we know how to do homotopy theory with **sSet**, so we should try taking the category of presheaves of simplicial sets.

Note: presheaf of sSet = simplicial object in Psh

Sheaves with Homotopy Type

Question

How do we associate a (pre)sheaf a homotopy type?

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From now on, the category $\mathrm{PShv}(Sm/S)$ will be the (∞-) category of simplicial presheaves over Sm/S.

Simplicial Presheaves

Model categorically: There is a projective model structure on $\mathrm{PShv}(Sm/S)$, with weak equivalence and fibration section-wise weak equivalence/fibration of simplicial sets. The fibrant objects are presheaves valued in Kan complexes.

$f: F \to G$ is a weak-equiv/fibration

$\Leftrightarrow$ $F(U) \to G(U)$ is weak-equiv/fibration of simplicial sets.

Simplicial Presheaves

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∞-**categorically**: $\mathrm{PShv}(\mathit{Sm}/S)$ is the ∞-category

$$\mathrm{PShv}(\mathit{Sm}/S) := \mathrm{Fun}((\mathit{Sm}/S)^{op}, \mathit{Spc})$$

where we view Sm/S as the trivial ∞-category by taking the nerve, and Spc is the ∞-category of spaces. $Spc := N(Kan)$

∞-Sheaves

Since we have presheaves valued in ∞-categories, we need more coherence condition for the descent data. For the rest of the discussion, we will fix the Nisnevich topology on Sm/S.

Definition

Given a cover $U := \{U_i \to X\}$, the **Cech Nerve** NU is the simplicial object

induced degeneracies

$$\ldots U \times_X U \times_X U \rightrightarrows U \times_X U \to U$$

Applying a presheaf $F \in \mathrm{PShv}(Sm/S)$ to the Cech nerve gives us a cosimplicial object $F(NU)$.

$U \times_X \cdots \times_X U$ = disjoint union of all n-ary fiber product U_i over X

Nisnevich Sheaf

Definition

A presheaf F is a **Nisnevich sheaf** if for every Nisnevich cover U, the induced map

$$F(X) \to lim_{\Delta} F(NU)$$

is an equivalence.

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Remark

We are taking the ∞-categorical limit here. If we were to use model categorical construction, we have to replace covers with **hypercovers** so that the Nisnevich sheaves will become the fibrant objects.

Sheafification via Localization

Time to break out your Higher Topos Theory: Let $\underbrace{\mathrm{Shv}_\tau(\mathcal{C})}_{\tau = Nis}$ be the ∞-category of τ-sheaf over some site $\mathcal{C}$.

Theorem

There exists a left exact localization functor

$$L_\tau : P\,Shv_\tau(\mathcal{C}) \to \cancel{P}\,Shv(\mathcal{C})$$

left adjoint to the inclusion functor.

1. Left exact $\Rightarrow$ preserves small limits

2. Left adjoint $\Rightarrow$ preserves all colimits

Examples

Example (Representables)

For $X \in Sm/S$, the represented presheaf

$$h_X := Hom(-, X)$$

of 0-dimensional simplicial sets. It is a Nisnevich sheaf since all higher coherence are automatic, and the Nisnevich topology is subcanonical.

Example (Constant Sheaves)

Let $Y \in Spc$ be fixed. Then the **constant sheaf** associated to Y is the sheafification of the constant presheaf valued at Y.

Theorem (Thomason-Trobaugh)

Algebraic K-theory is a Nisnevich sheaf.

Simplification

We have a simple criterion to check when a presheaf is a Nisnevich sheaf.
A pullback diagram

Grothendieck cd-structure

$$\begin{tikzcd} K(U \times_X V) & K(V) \arrow[l] \\ K(U) \arrow[u] & K(X) \arrow[l] \arrow[u] \end{tikzcd}$$

induces

Mayer-Vietoris

LES

$$\begin{tikzcd} U \times_X V \arrow[r] \arrow[d] & V \arrow[d, "p"] \\ U \arrow[r, "i"'] & X \end{tikzcd}$$

is called a **Nisnevich square** if i is an open immersion, p is étale, and p restricts to a isomorphism $p^{-1}(X - U) \to X - U$. ← w/ reduced scheme structure

Theorem

A presheaf F is a Nisnevich sheaf iff it satisfies the following critera

1. $F(\emptyset) = *$
2. *F sends every Nisnevich distinguished square to a homotopy pullback.*

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$\mathbb{A}^1$-Locality

Quote

All our constructions are based on the intuitive feeling that ... there should exist a homotopy theory of algebraic varieties where the affine line plays the role of the unit interval.

-Morel, Voevodsky

$\mathbb{A}^1$-Locality

Definition

A presheaf $\mathcal{F} \in \mathrm{PShv}(Sm/S)$ is called $\mathbb{A}^1$-**invariant** (or $\mathbb{A}^1$-**local**) if the canonical projection map

$$X \times \mathbb{A}^1 \to X$$

induces an equivalence

$$F(X) \to F(X \times \mathbb{A}^1)$$

for all S-scheme X.

Examples

Theorem (Quillen-Suslin)

Let $X = Spec(R)$ where R is a regular k-algebra. Then

$$Vect(X) \to Vect(X \times \mathbb{A}^1)$$

is an equivalence.

Theorem (Algebraic K-theory)

Let X be in Sm/S. Then, Fundamental Theorem of K-theory

$$K(X) \to K(X \times \mathbb{A}^1)$$

is an equivalence.

The above theorem does not hold when X is singular, which is the reason why we restrict to smooth schemes.

Non-example

Easy Exercise

The presheaf represented by $\mathbb{A}^1$ is not $\mathbb{A}^1$-local.

$\mathbb{A}^1$ represents global section

$$\mathbb{A}^1(\mathbb{A}^1) \to \mathbb{A}^1(\mathbb{A}^1 \times \mathbb{A}^1)$$

not an equivalence

$\mathbb{A}^1$-Localization

Since we want to study study schemes in Sm/S, we need a localization functor

$$L_{\mathbb{A}^1} : \mathrm{PShv}(Sm/S) \to \mathrm{PShv}_{\mathbb{A}^1}(Sm/S)$$

Note that the condition of $\mathbb{A}^1$-invariance is equivalent to being local to the set of maps

Noetherian + finite type

$$S := \{X \times \mathbb{A}^1 \to X : X \in Sm/S\}$$

$\mathrm{Hom}(X, F) \to \mathrm{Hom}(X \times \mathbb{A}^1, F)$ is an equiv.

$\rightsquigarrow F(X) \to F(X \times \mathbb{A}^1)$

Theorem (HTT 5.5.4.15)

If $\mathcal{C}$ is presentable and $S \subset Mor\mathcal{C}$ is small, then the inclusion of the full subcategory of S-local objects admits a left adjoint.

In particular, the $\mathbb{A}^1$-localization functor exists.

1. preserve finite limits

2. preserves colimits

We can give an explicit formula for $\mathbb{A}^1$-localization:

Definition

The **algebraic n-simplex** is the following scheme

$$\Delta^n := Spec(\mathbb{Z}[t_1, ..., t_{n+1}]/(\sum t_i - 1))$$

$\Delta^n_{Top} = \{$ sum of coordinates are 1 in $\mathbb{R}^{n+1} \}$

Definition

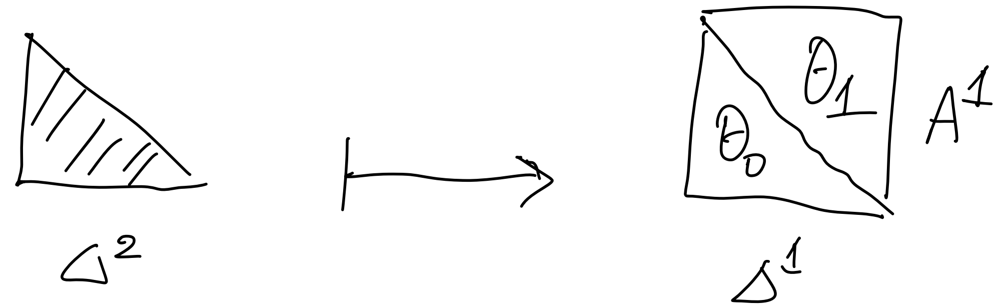
The **singular chains** construction

$$Sing : \mathrm{PShv}(Sm/S) \to \mathrm{PShv}_{\mathbb{A}^1}(Sm/S)$$

is defined by

$$Sing(F)(X) := colim_{\Delta^{op}}(F(X \times \Delta^n))$$

Sing functor



Proposition

Sing(F) is $\mathbb{A}^1$-local for any presheaf F. MV, MVW

Corollary

Sing(F) *is equivalent to the localization functor* $L_{\mathbb{A}^1}$.

Outline: lemma: $F(X) \to F(X \times \mathbb{A}^1)$ is an equivalence iff $i_0^*, i_1^* : F(X \times \mathbb{A}^1) \to F(X)$ are homotopic

Produced a map $\theta_k : \Delta^{n+1} \to \Delta^n \times \mathbb{A}^1$ ⇝ induced desired homotopy

$$v_i \mapsto \begin{cases} v_i \times \{0\} & i \leq k \\ v_i \times \{1\} & \text{otherwise} \end{cases}$$

Problem

Nisnevich Sheafification may break $\mathbb{A}^1$-locality; $\mathbb{A}^1$-localizating a Nisnevich sheaf may break the sheaf condition.

MV 3.2.7

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Motivic Spaces

Definition

The ∞-category of **motivic spaces** $Spc(S)$ is

$$Spc(S) = \mathrm{Shv}_{\mathrm{Nis}}(Sm/S) \cap \mathrm{PShv}_{\mathbb{A}^1}(Sm/S) \subseteq \mathrm{PShv}(Sm/S)$$

which is the full subcategory of presheaves that are Nisnevich sheaves and are $\mathbb{A}^1$-local.

Examples

$\mathbb{G}_m$ represent units

$\mathbb{G}_m(X) = \Gamma(X, \mathcal{O}_X^*)$

$R[t]^* \longrightarrow R^*$

equivalence

Example

The representable $\mathbb{G}_m$ is $\mathbb{A}^1$-invariant, since it represents units. Thus, it is a motivic space.

Theorem

Algebraic K-theory is a motivic space.

Definition

The **motivic localization functor**

$$L_{mot} : \mathrm{PShv}(Sm/S) \to Spc(S)$$

is defined to be the colimit in the presheaf category

$$L_{mot} : colim(L_{Nis} \to L_{\mathbb{A}^1} L_{Nis} \to L_{Nis} L_{\mathbb{A}^1} L_{Nis}) \to ...$$

To see that this indeed lands in $Spc(S)$, we can look at two cofinal sequences in the colimit.

$$\mathrm{colim}\left(L_{\mathbb{A}^1} L_{Nis} \to (L_{\mathbb{A}^1} L_{Nis})^2 \to (L_{\mathbb{A}^1} L_{Nis})^3 \to \dots\right)$$

$\Rightarrow$ $\mathbb{A}^1$-local

$$\mathrm{colim}\left(L_{Nis} \to L_{Nis}(L_{\mathbb{A}^1} L_{Nis}) \to L_{Nis}(L_{\mathbb{A}^1} L_{Nis})^2 \to \dots\right)$$

$\Rightarrow$ Nisnevich sheaf

Definition

We say that $f : F \to G$ in $\mathrm{PShv}(Sm/S)$ is a **motivic equivalence** if it becomes an equivalence after motivic localization.

Example

The map to the terminal object

$$\mathbb{A}^n_S \to S$$

is a motivic equivalence for all $n \geq 1$.

$\mathbb{A}^1$ - by definition

$\mathbb{A}^n$ - holds L_{mot} finite product

More Examples

Proposition

For any presheaf $F \in \mathrm{PShv}(Sm/S)$, the projection map

$$F \times \mathbb{A}^n_S \to F$$

is a motivic equivalence.

For representables, also by definition.

Fact: all presheaves are colimits of representables

$$F \cong \operatorname{colim} h_X$$

∞-cat Fact: colimit distribute over product in our case

$$\begin{array}{c} F \times A^n \to F \\ \uparrow \\ \operatorname{colim}(h_X) \times A^n \\ \downarrow \\ \operatorname{colim}(h_X \times A^n) \end{array}$$

Pointed Motivic Spaces

Definition

A motivic space X is **pointed** if it is equipped with a map from the terminal object S. We denoted the category of pointed motivic spaces as $Spc_*(S)$, with zero object the basepoint $*$.

There is the usual adjunction

$$Spc(S) \leftrightarrows Spc_*(S)$$

by adjoining a distinct basepoint and forgetting the basepoint.

$$X \amalg * =: X_+$$

Constructions

Definition

The **cofiber** of a map between two pointed motivic spaces $f : Y \to X$, denoted by X/Y, is the pushout

$$\begin{tikzcd} Y \arrow[r] \arrow[d] & X \arrow[d] \\ * \arrow[r] & X/Y \end{tikzcd}$$

Definition

The **smash product** of two motivic spaces X, Y is defined to be the cofiber of the canonical map $X \vee Y \to X \times Y$

$$X \wedge Y := X \times Y/X \vee Y$$

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The Spheres

Since we are mixing schemes with simplicial sets, there are two types of spheres in motivic homotopy theory.

Definition

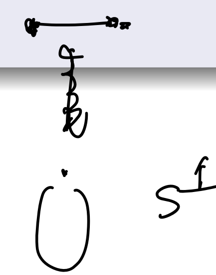
The multiplicative group $\mathbb{G}_m$ pointed at 1 is called the **Tate sphere**, denoted by $S^{1,1}$.

$\| \; \mathbb{A}^1 - \{0\}$

Topological $\mathbb{A}^1_{\mathbb{C}} - \{0\} \overset{\mathbb{C}}{\simeq} \mathbb{C} - \{0\} \simeq S^1$

Definition

The constant presheaf at the simplicial circle $S^1 := \Delta^1 / \partial\Delta^1$ is the **simplicial sphere**, denoted by $S^{1,0}$.



$S^{1,1} \wedge S^{1,0} =: S^{2,0}$

Smash Product

Definition

The **suspension** of a pointed motivic space X is defined to be the pushout

$$\begin{tikzcd} X \arrow[r] \arrow[d] & * \arrow[d] \\ * \arrow[r] & \Sigma X \end{tikzcd}$$

Topologically

$S^1 \wedge X \cong \Sigma X$

Proposition

There is a canonical motivic equivalence

$$\Sigma X \cong S^{1,0} \wedge X$$

Exercise:

$\Sigma X(U) \cong S^1 \wedge X(U) \quad \forall U$

Outline: L_{mot} preserves colimits (wedges & quotients) and finite product so it suffices to show equivalence on the level of simplicial presheaves.

Examples

Thm (Asok, Doran, Fasel) $S^{a,b}$ is not motivic equiv to a smooth scheme for $a > 2b$

There a few explicit descriptions of smashing the two kinds of motivic spheres together, but we do have a class of specific examples

Example

We have the canonical equivalence

$$\Sigma \mathbb{G}_m \cong \mathbb{P}^1$$

$$\begin{tikzcd} \mathbb{G}_m \arrow[r] \arrow[d] & \mathbb{A}^1 \arrow[d] \\ \mathbb{A}^1 \arrow[r] & \mathbb{P}^1 \arrow[ul, phantom, "\ulcorner", very near start] \end{tikzcd} \quad \rightsquigarrow \quad \begin{tikzcd} \mathbb{G}_m \arrow[r] \arrow[d] & * \arrow[d] \\ * \arrow[r] & \mathbb{P}^1 \cong \Sigma \mathbb{G}_m \end{tikzcd}$$

Examples

$n=1$, nothing

Vertically

$$\begin{tikzcd} \mathbb{G}_m \wedge \mathbb{G}_m \arrow[r] \arrow[d] & * \arrow[d] \\ * \arrow[r] & \Sigma(\mathbb{G}_m \wedge \mathbb{G}_m) \end{tikzcd}$$

Horizontally

$$\begin{tikzcd} * \arrow[r] \arrow[d] & \mathbb{A}^2 - \{0\} \\ * & \end{tikzcd}$$

Example

We have the canonical equivalence

$$\mathbb{A}^n - \{0\} \overset{mot}{\cong} (S^1)^{\wedge(n-1)} \wedge (\mathbb{G}_m)^{\wedge n} = S^{2n-1,n}$$

$n=2$, WTS $\mathbb{A}^2 - \{0\} \overset{mot}{\cong} \Sigma(\mathbb{G}_m)^{\wedge 2}$

$$\begin{tikzcd} \mathbb{G}_m \times \mathbb{G}_m \arrow[r] \arrow[d] & \mathbb{G}_m \times \mathbb{A}^1 \arrow[d] \\ \mathbb{G}_m \times \mathbb{A}^1 \arrow[r] & \mathbb{A}^2 - \{0\} \arrow[ul, phantom, "\ulcorner", very near start] \end{tikzcd}$$

is a pushout.

$$\wr$$

$$\begin{tikzcd} \mathbb{G}_m \times \mathbb{G}_m \arrow[r] \arrow[d] & \mathbb{G}_m \arrow[d] \\ \mathbb{G}_m \arrow[r] & \mathbb{A}^1 \end{tikzcd}$$

$$\begin{tikzcd} * & * \arrow[l] \arrow[r] & * \\ \mathbb{G}_m \arrow[u] \arrow[d] & \mathbb{G}_m \vee \mathbb{G}_m \arrow[l] \arrow[r] \arrow[u] \arrow[d] & \mathbb{G}_m \arrow[u] \arrow[d] \\ \mathbb{G}_m & \mathbb{G}_m \times \mathbb{G}_m \arrow[l] \arrow[r] & \mathbb{G}_m \end{tikzcd}$$

$\mathbb{A}^1$-Homotopy Sheaves

We can now define the motivic analog of homotopy groups, which are now Nisnevich sheaves.

Definition

The $\mathbb{A}^1$-**homotopy sheaf** of a pointed motivic space (X, x), denoted by $\pi_n^{\mathbb{A}^1}(X, x)$, is the Nisnevich sheafification of the presheaf

$$U \mapsto [\Sigma^n U_+, X]_{Spc(S)_*}$$

$$[X, Y]_C = \pi_0 \operatorname{Maps}_C(X, Y)$$

The $\mathbb{A}^1$-**homotopy sheaf** can be computed in the following way:

Definition

Let (X, x) be a pointed Nisnevich sheaf. Let $\pi_n^{\mathrm{Nis}}(X, x)$ be the **Nisnevich homotopy sheaf**, defined to be the Nisnevich sheafification of the presheaf

$$U \mapsto \pi_n(X(U), x)$$

$\pi_n(|X(U)|, x)$

Proposition

If (X, x) is a motivic space, then the Nisnevich homotopy sheaf and $\mathbb{A}^1$-homotopy sheaf of (X, x) agree.

$$\pi_n^{\mathrm{Nis}}(X, x) \cong \pi_n^{\mathbb{A}^1}(X, x)$$

Whitehead's Theorem

Theorem (Whitehead's Theorem)

Let $f : F \to G$ be a map in $PShv(Sm/S)$. Then, f is a motivic equivalence iff

$$\pi_n^{\mathbb{A}^1}(f) : \pi_n^{\mathbb{A}^1}(F, x) \to \pi_n^{\mathbb{A}^1}(G, f(x))$$

is an equivalence for all $n \geq 0$ and all basepoint $x \in F$.

Follows from Whitehead's Theorem $\pi_n^{Nis}(X, x)$

+

equivalence

$\pi_n^{A^1} \cong \pi_n^{Nis}$

More to come:

1. Check motivic equiv on affines
2. Fibre sequence + LES of homotopy sheaves
3. Thom space & purity
4. Eilenberg-Maclane spaces (motivic analog) representability.